For all questions, answer choice (E) NOTA means that none of the given answers is correct. For all problems, $i = \sqrt{-1}$.
Good Luck!

1. Which of the following lines is parallel to $123x + 456y = 789$?
   (A) $y = -\frac{123}{456}x + 1$ (B) $y = -\frac{456}{123}x$ (C) $y = \frac{123}{456}x + 7$ (D) $y = -\frac{456}{123}x + 3$ (E) NOTA

2. In order to annoy every mathematician, Akhil uses the variables $e = 2$ and $\pi = 3$. What value should he get when he evaluates $\pi^e - e^\pi$ using only the values of Akhil's variables and not the actual values of the constants? (the answers are also in terms of these variables)
   (A) $\pi - e$ (B) $-\pi$ (C) $e - \pi$ (D) 0 (E) NOTA

3. You find a note that reminds you that a tower of exponents should be solved from top to bottom. The note also asks you to simplify $\sqrt{2^{6^{2^{1^{4^4}}}}}$. What is the positive value of the expression from the note?
   (A) 4096 (B) 65536 (C) 262144 (D) 2^{128} (E) NOTA

4. Let $d(N)$ denote the number of positive integral divisors of a positive integer N. Let $r > 1$ be the least positive integer that satisfies $\frac{d(r+1)}{d(r-1)} = 4$. Compute $d(r)$.
   (A) 4 (B) 5 (C) 6 (D) 8 (E) NOTA

5. What is the value of $\log_2 8^3 - (\log_2 8)^3$?
   (A) -3 (B) 0 (C) -18 (D) 18 (E) NOTA

6. Factor $1 - 4x^2 - 12xy - 9y^2$.
   (A) $(1 - 2x - 3y)(1 - 2x + 3y)$
   (B) $(1 - 2x + 3y)(1 + 2x - 3y)$
   (C) $(1 - 2x + 3y)(1 - 2x + 3y)$
   (D) $(1 - 2x - 3y)(1 + 2x + 3y)$
   (E) NOTA

7. Solve for x: $x^2 + 12 = x + 12$
   (A) 2, 3 (B) 0 (C) 0, 1 (D) $0, i\sqrt{12}$ (E) NOTA

8. Evaluate $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$.
   (A) $\begin{bmatrix} 13 & 20 \\ 7 & 10 \end{bmatrix}$ (B) $\begin{bmatrix} 4 & 6 \\ 3 & 8 \end{bmatrix}$ (C) $\begin{bmatrix} 6 & 7 \\ 16 & 17 \end{bmatrix}$ (D) $\begin{bmatrix} 6 & 4 \\ 12 & 17 \end{bmatrix}$ (E) NOTA

9. Which of the following is equal to $\frac{5i+4}{\sqrt{3}+2+i}$?
   (A) $\frac{4i\sqrt{3}-3i-5\sqrt{3}+14}{4}$ (B) $\frac{3i\sqrt{3}-5i-5\sqrt{3}+12}{2}$ (C) $\frac{8i\sqrt{3}-7i-3\sqrt{3}+14}{8}$ (D) $\frac{12i\sqrt{3}-3i-6\sqrt{3}+8}{4}$ (E) NOTA

10. Evaluate:
$$\prod_{n=0}^{\infty} i^{2^{-n}}.$$
   (A) i (B) -1 (C) $-i$ (D) i^e (E) NOTA

11. Ananya's name has the letter "n" two times so she wants you to solve for n in the equation $n^2 + 2^2n + 2^2 = 2^n$. Find the sum of all integer values of n that satisfy the equation.

(A) 0 (B) 2 (C) 6 (D) 12 (E) NOTA

12. Karthik's favorite type of expression, not-a-polynomial, is in the form of $ax^2 + bx + c + dx^{-1}$. What are the roots of the not-a-polynomial where $a = 1$, $b = 2$, $c = 3$, and $d = 6$?

(A) $2i$ (B) $-2, 3, 6$ (C) $1, \pm\sqrt{6}i$ (D) $-2, \pm\sqrt{3}i$ (E) NOTA

13. Compute

$$\sum_{n=1}^{10} \begin{vmatrix} n+1 & n+3 & n+5 \\ n+7 & 0 & n+9 \\ n+11 & n+13 & n+15 \end{vmatrix}$$

(A) 5400 (B) 16000 (C) 2200 (D) 3600 (E) NOTA

14. Eric and Dylan are playing a game where they try not to laugh at memes (curated by Tanmay). There is a 70% chance that Eric will not laugh at a meme. On the other hand, there is a 60% chance that Dylan will not laugh at a meme. Each round consists of showing Eric and then Dylan a meme. The person who laughs first loses the game and the game ends instantly. What is the probability that Dylan wins this difficult game?

(A) $\frac{14}{29}$ (B) $\frac{15}{29}$ (C) $\frac{2}{5}$ (D) $\frac{3}{5}$ (E) NOTA

15. Call a positive integer q to be *decomposable* if there exist positive integers a, b such that $\sqrt{a} + \sqrt{b} = \sqrt{q}$. Which of the following numbers are decomposable?

(A) 2021 (B) 2022 (C) 2023 (D) 2026 (E) NOTA

16. Jenna took a survey and found that 50% of the people from her school are taking geometry, 30% are taking Algebra II, and 10% are taking both geometry and Algebra II. What percentage of people in the school are taking only one of the two math classes?

(A) 50% (B) 60% (C) 70% (D) 80% (E) NOTA

17. Sruthi likes to find cube roots of perfect cubes in her free time. What are the non-integer values of x that satisfy the equation $x^3 = 27$?

(A) $\frac{3}{2} \pm \frac{3i\sqrt{6}}{4}$ (B) $\frac{3}{4} \pm \frac{i\sqrt{3}}{3}$ (C) $\frac{-3}{2} \pm \frac{9i\sqrt{3}}{2}$ (D) $\frac{-3}{2} \pm \frac{3i\sqrt{3}}{2}$ (E) NOTA

18. Given that $f(x)$ is a cubic function, what is the value of $f(4)$ if $f(0) = 1$, $f(1) = 2$, $f(2) = 4$, and $f(3) = 9$?

(A) 19 (B) 17 (C) 16 (D) 9 (E) NOTA

19. Find $A + B + C$ if the following equation is true for all $x \neq 9$.

$$\frac{7x-4}{(x-9)^3} = \frac{A}{x-9} + \frac{B}{(x-9)^2} + \frac{C}{(x-9)^3}$$

(A) 36 (B) 56 (C) 66 (D) 81 (E) NOTA

20. Evaluate $\displaystyle\sum_{a=2}^{\infty} \Big(\sum_{b=2}^{\infty} \frac{1}{a^b} + \sum_{b=2}^{\infty} \frac{1}{(-a)^b} \Big)$.

(A) 0.25 (B) 0.5 (C) 1 (D) 1.5 (E) NOTA

21. Find the value of
$$\sum_{x=1}^{\infty} \frac{x^2}{4^x}$$
(A) $\frac{4}{3}$ (B) $\frac{3}{4}$ (C) $\frac{25}{28}$ (D) $\frac{20}{27}$ (E) NOTA

22. Let $f(x)$ and $g(x)$ be functions with domain $\mathbb{N}$ (not including 0). These functions satisfy $f(x) = f(x-1) + g(x)$, $f(1) = 2$, and $g(x) = \binom{x}{x} + \binom{x}{x-1} + ... + \binom{x}{1} + \binom{x}{0}$ where $\binom{n}{r} = \frac{n!}{r!(n-r)!}$. What is the value of $f(64)$?
(A) $2^{127} - 1$ (B) $2^{65} - 1$ (C) $2^{65} - 2$ (D) $2^{127} - 2$ (E) NOTA

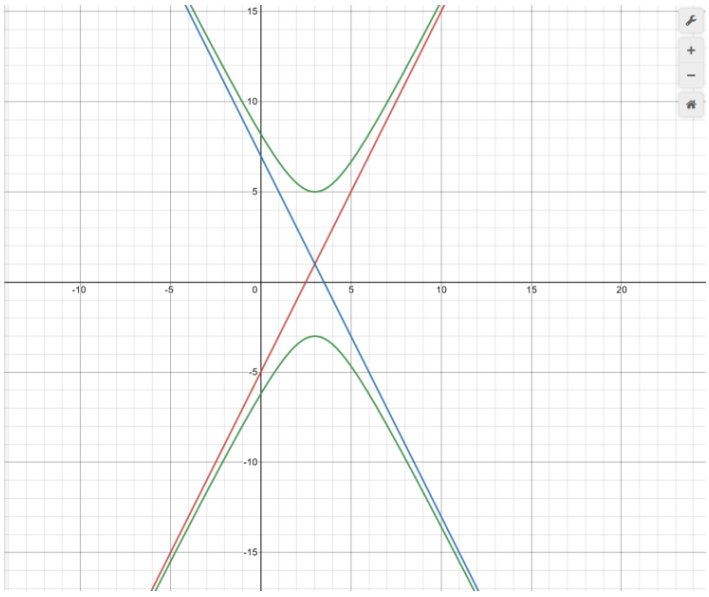
23. Find the area enclosed by the conic represented in the complex plane by $|z - 1 - i| + |z + 2 + 3i| = 13$.
(A) 39π (B) 40π (C) 42π (D) 45π (E) NOTA

24. What is the slope of the tangent line which intersects $(x-3)^2 + (y-4)^2 = 25$ at (a, b), a point on this circle, given that this slope is defined?
(A) $\frac{3-a}{b-4}$ (B) $\frac{b+4}{a-3}$ (C) $\frac{b-4}{a+3}$ (D) $\frac{a-4}{b-3}$ (E) NOTA

25. The point $(0, 1)$ is reflected across the line $y = ax$, and the resulting point lies on the line $y = x$. Find $\lfloor 10a \rfloor$.
(A) 14 (B) 17 (C) 20 (D) 24 (E) NOTA

26. Shubham's swimming pool is an ellipse defined by the equation $x^2 + 4y^2 - 6x - 8y + 9 = 0$. If he is currently treading water at the point $(2, 1)$, what is the shortest distance he must travel to reach the pool's edge?
(A) $\frac{\sqrt{6}}{3}$ (B) $\frac{\sqrt{3}}{3}$ (C) $\frac{2\sqrt{3}}{3}$ (D) $\frac{\sqrt{3}}{2}$ (E) NOTA

27. Nitish loves graphing hyperbolas to solve problems regardless of the presence of hyperbolas in the problem. Knowing that the lines are the asymptotes of the hyperbola, which of the following is an equation of the hyperbola shown in the graph below?



(A) $\frac{(y-1)^2}{2} + \frac{(x-3)^2}{4} = 1$ (B) $\frac{(y-1)^2}{4} + \frac{(x-3)^2}{16} = 1$ (C) $\frac{(y-1)^2}{8} - \frac{(x-3)^2}{4} = 1$ (D) $\frac{(y-1)^2}{16} - \frac{(x-3)^2}{4} = 1$ (E) NOTA

28. Consider the set $A = \{x-1, x-2, x-3, x-4, x-5, x-6, x-7, x-8\}$. Let B be a random 4-element subset of A, and let C denote the complement of B with respect to A. Let

$$f(B) = \prod_{b \in B} b - \prod_{c \in C} c$$

What is the expected value of the degree of $f(B)$?

(A) $\frac{20}{7}$ (B) $\frac{29}{10}$ (C) $\frac{41}{14}$ (D) 3 (E) NOTA

29. For any sequence $A = (a_1, a_2, a_3 \cdots)$, let $f(A)$ be the value of the series $\frac{1}{a_1} + \frac{1}{a_2^2} + \frac{1}{a_3^3} + \cdots$ if it converges, or "DNE" if the series does not converge or is not defined. For any pair of real numbers, (a, d), let $B_{a,d}$ denote the sequence $(a, a+d, a+2d, a+3d, \cdots)$. Call such a pair (a, d) "lethal" if $f(B_{a,d}) =$ "DNE". Compute the maximum value of ad over all lethal pairs (a, d).

(A) -1 (B) 0 (C) 1 (D) ∞ (E) NOTA

30. After writing this test, Shrung realized that he forgot to put a question about absolute values and mathematical casework. How many lattice points (points whose x-coordinates and y-coordinates are integers) are inside the area bounded by $y = |x|$ and $y = -x^2 + 8$?

(A) 10 (B) 19 (C) 12 (D) 21 (E) NOTA